



# **Ikeldilen argumentiň trigonometrik funksiýalary**



# Barlagnama:

1.  $\cos(\alpha - \beta) = ?$

- a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$
- b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$
- c)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$
- d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

2.  $\cos(\alpha + \beta) = ?$

- a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$
- b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$
- c)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$
- d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

3.  $\sin(\alpha + \beta) = ?$

- a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$
- b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$
- c)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$
- d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

4.  $\sin(\alpha - \beta) = ?$

- a)  $\cos \alpha \cos \beta - \sin \alpha \sin \beta$
- b)  $\cos \alpha \cos \beta + \sin \alpha \sin \beta$
- c)  $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$
- d)  $\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$



5.  $\operatorname{tg}(\alpha + \beta) = ?$

a)  $\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta}$

b)  $\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}$

c)  $\frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta}$

d)  $\frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}$

6.  $\operatorname{tg}(\alpha - \beta) = ?$

a)  $\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta}$

b)  $\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}$

c)  $\frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta}$

d)  $\frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}$

7.  $\operatorname{Ctg}(\alpha - \beta) = ?$

a)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta + 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$

b)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha - \operatorname{ctg} \beta}$

c)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta + 1}{\operatorname{ctg} \alpha - \operatorname{ctg} \beta}$

d)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$

8.  $\operatorname{Ctg}(\alpha + \beta) = ?$

a)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta + 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$

b)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha - \operatorname{ctg} \beta}$

c)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta + 1}{\operatorname{ctg} \alpha - \operatorname{ctg} \beta}$

d)  $\frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$

1	2	3	4	5	6	7	8
b	a	ç	d	a	d	ç	d




$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

Goý,  $\beta = \alpha$  bolsun, onda

$$\begin{aligned}\sin 2\alpha &= \sin(\alpha + \alpha) = \\ &= \sin \alpha \cdot \cos \alpha + \cos \alpha \cdot \sin \alpha = 2 \sin \alpha \cos \alpha\end{aligned}$$


$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Goý,  $\beta = \alpha$  bolsun, onda

$$\begin{aligned}\cos 2\alpha &= \cos(\alpha + \alpha) = \\ &= \cos \alpha \cos \alpha - \sin \alpha \sin \alpha = \cos^2 \alpha - \sin^2 \alpha\end{aligned}$$

$$\cos^2 \alpha - \sin^2 \alpha = 1 - \sin^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha$$

$$\begin{aligned}\cos^2 \alpha - \sin^2 \alpha &= \cos^2 \alpha - (1 - \cos^2 \alpha) = \cos^2 \alpha - 1 + \cos^2 \alpha = \\ &= 2 \cos^2 \alpha - 1\end{aligned}$$





$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$$

$$\beta = \alpha$$

$$\operatorname{tg}(\alpha + \alpha) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\alpha}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\alpha}$$

$$\operatorname{tg}2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$$



Berlen  $\cos x = 0,8$ ;  $0 < x < \frac{\pi}{2}$ . Tapmaly:  $\sin 2x$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin x = \pm \sqrt{1 - \cos^2 x} = \pm \sqrt{1 - 0,8^2} =$$

$$= \pm \sqrt{1 - 0,64} = \pm \sqrt{0,36} = \pm 0,6$$

$$\sin x = 0,6$$

$$\sin 2x = 2 \sin x \cdot \cos x = 2 \cdot 0,8 \cdot 0,6 = 0,96$$



Aňlatmany ýönekeýleşdiriň:  $\frac{\sin 40^\circ}{\sin 20^\circ}$

$$\frac{\sin 40^\circ}{\sin 20^\circ} = \frac{\sin(2 \cdot 20^\circ)}{\sin 20^\circ} = \frac{2 \sin 20^\circ \cdot \cos 20^\circ}{\sin 20^\circ} = 2 \cos 20^\circ$$



# Temany berkitmek:

103-nji gönükmek.

$\cos \alpha = \frac{1}{2}$  we  $0 < \alpha < \frac{\pi}{2}$  bolsa, onda  $\sin 2\alpha, \cos 2\alpha, \operatorname{tg} 2\alpha$  hasaplaň

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha} = \pm \sqrt{1 - \left(\frac{1}{2}\right)^2} = \pm \sqrt{1 - \frac{1}{4}} = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$$0 < \alpha < \frac{\pi}{2}, \text{onda } \sin \alpha = \frac{\sqrt{3}}{2}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$\operatorname{tg} 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$



## 106-njy gönükmə.

a) 
$$\frac{(\sin\alpha + \cos\alpha)^2}{1 + \sin 2\alpha} = \frac{\sin^2 \alpha + 2\sin\alpha\cos\alpha + \cos^2 \alpha}{1 + \sin 2\alpha} =$$
$$= \frac{1 + \sin 2\alpha}{1 + \sin 2\alpha} = 1$$

b) 
$$\frac{\cos 2\alpha}{\sin\alpha + \cos\alpha} = \frac{\cos^2 \alpha - \sin^2 \alpha}{\sin\alpha + \cos\alpha} =$$
$$= \frac{(\cos \alpha + \sin \alpha) \cdot (\cos \alpha - \sin \alpha)}{\sin \alpha + \cos \alpha} = \cos \alpha - \sin \alpha$$



# Öý işi

- 107-nji gönükmə.

