Simplification

$$YX + X = X$$

$$XY + X\overline{Y} = X$$

2nd Distributive

$$(X + A)(X + B) = X + AB$$

$$(X + A)(X + B)(X + C) = X + ABC$$

Absorption

$$Y + X\overline{Y} = X + Y$$

Most Common Stupid Errors

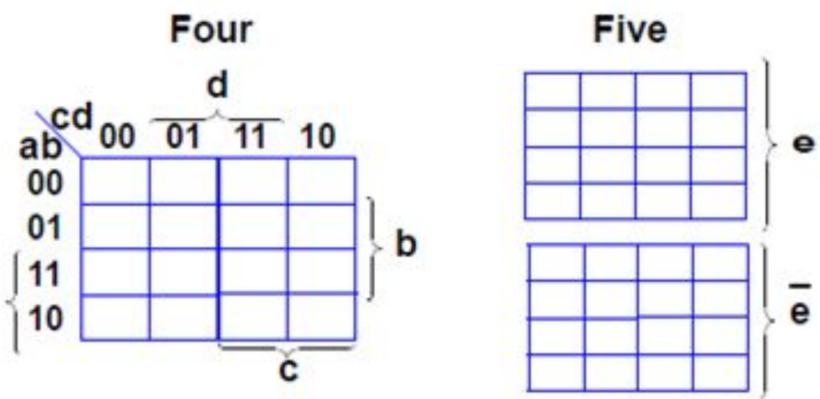
$$\cancel{\overline{X}\cdot\overline{Y} = \overline{XY}}$$

$$\cancel{X + 1 = X}$$

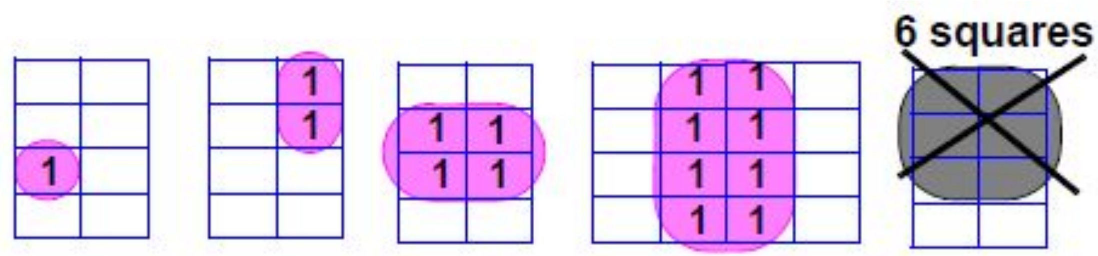
Lecture 4: Multiple Output Maps

senior-lecturer Kairat Sariyev, email: ksariyev@gmail.com, room 409

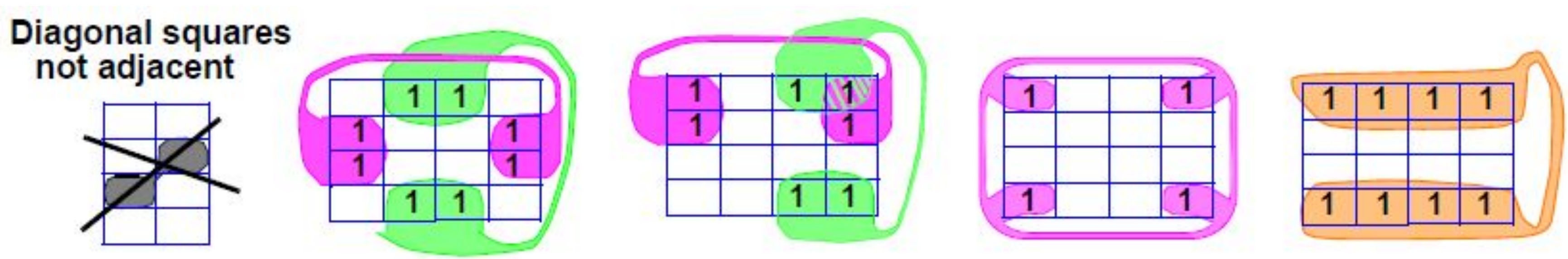
Review



Four
Five
d
cd
ab
00 01 11 10
00
01
11
10
b
c
e
ē



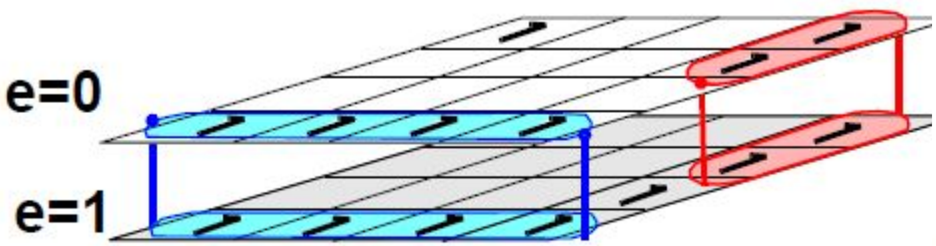
6 squares

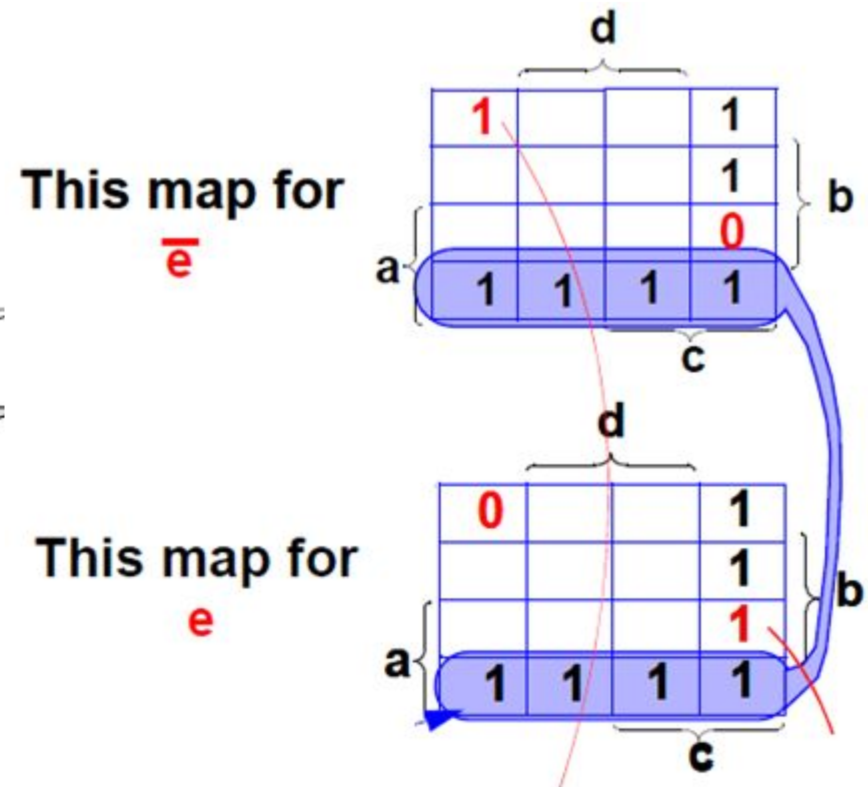


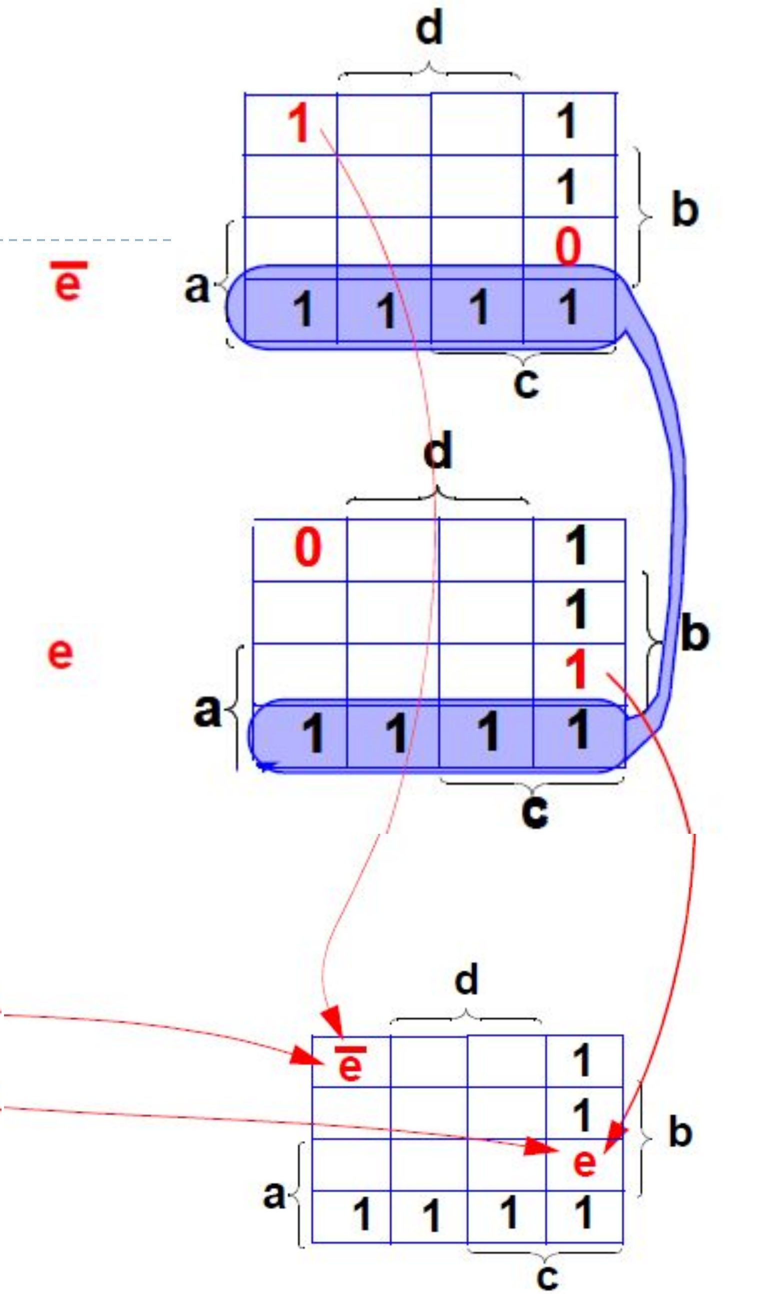
Diagonal squares
not adjacent

Five-Variable Karnaugh Maps

Think of two maps on top of each other







Can loop: e 1 | ē 1 | 1 1

Cannot loop: ~~ē e~~

"e" for "1"s on top layer

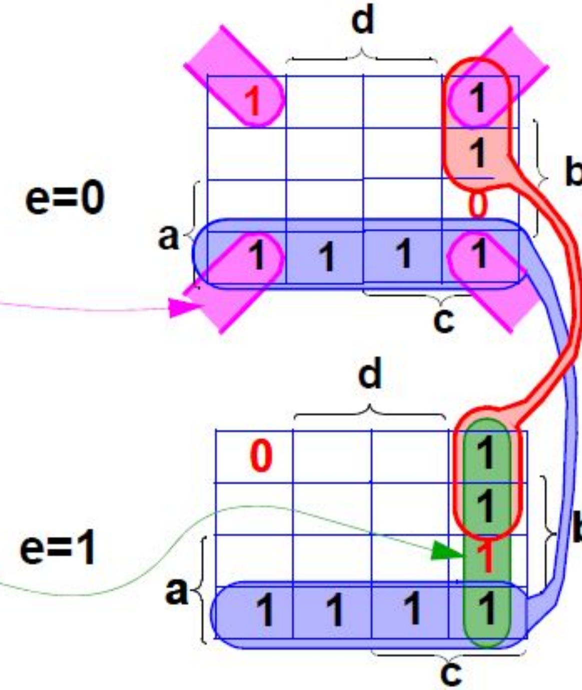
"ē" for "1"s on bottom layer.

Equations From Five Variable Maps

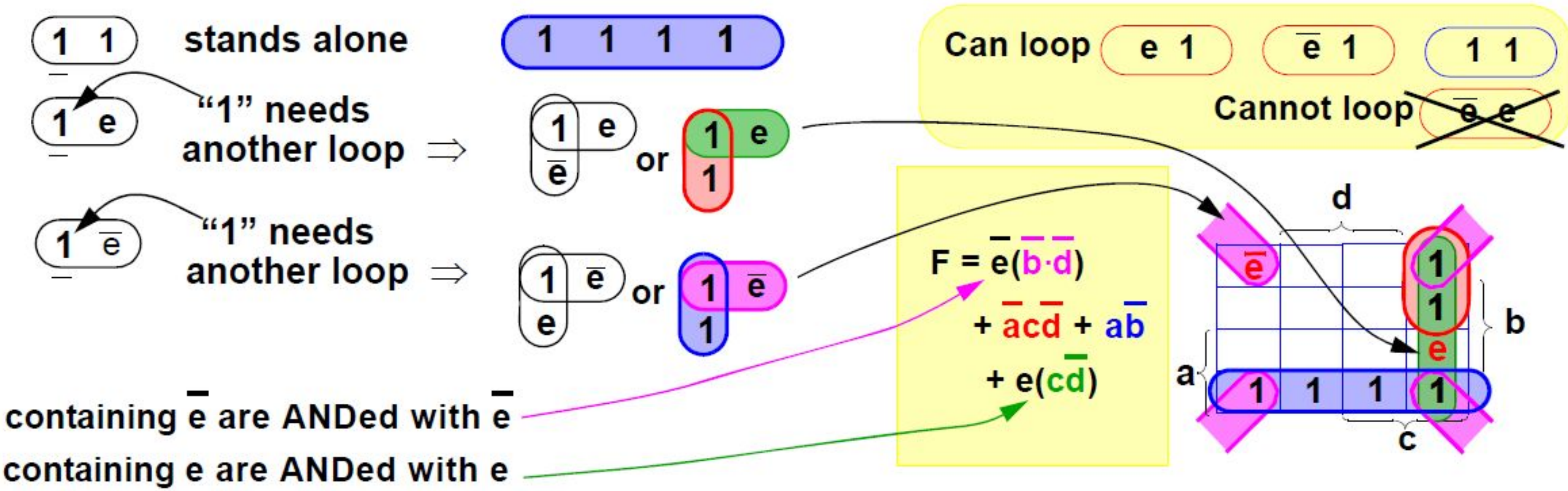
e=0 (top) map are ANDed with $\bar{e}$

e=1 (bottom) map are ANDed with e

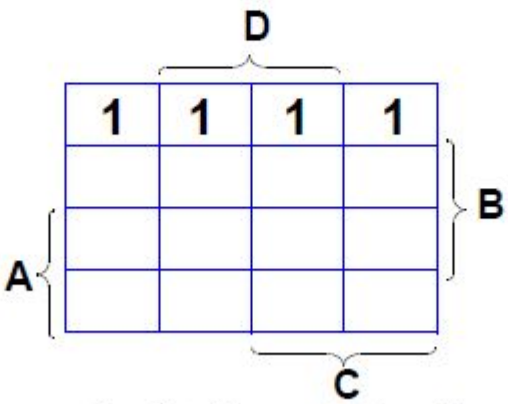
$$F = \bar{e}(\bar{b}\cdot\bar{d}) + \bar{a}c\bar{d} + a\bar{b} + e(c\bar{d})$$



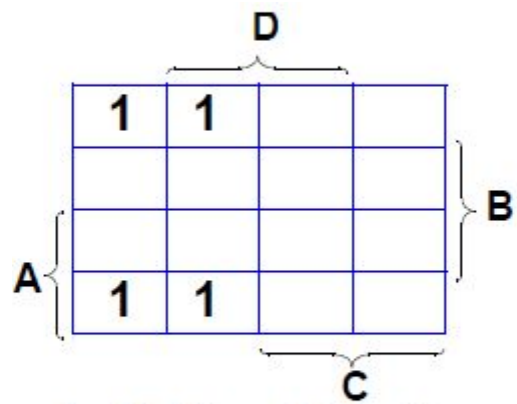
Equations From Five Variable Maps



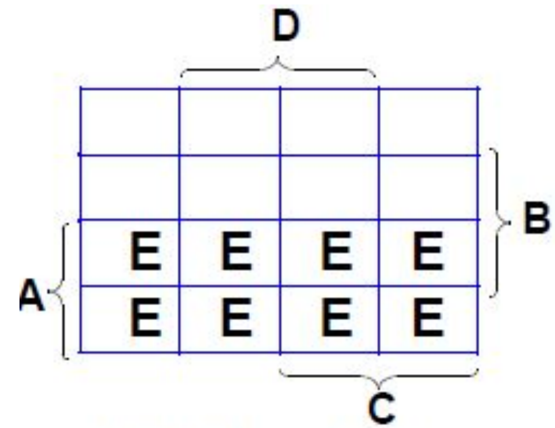
Example: F is true when:
both A and B are off
or both C and B are off
or both A and E are on



both A and B off

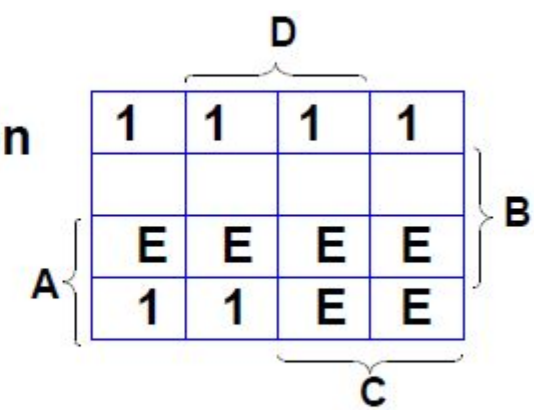


both C and B off



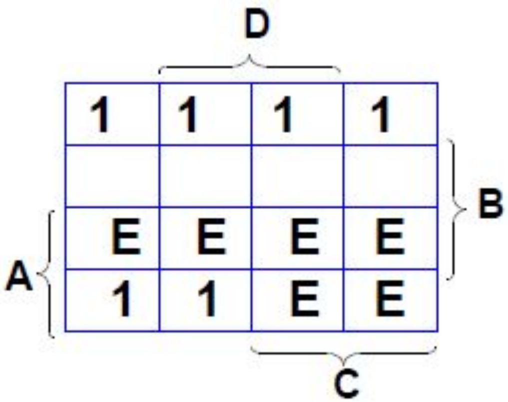
both A and E on

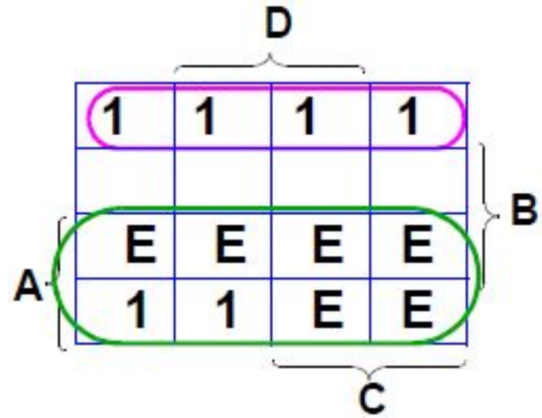
Combine on one map



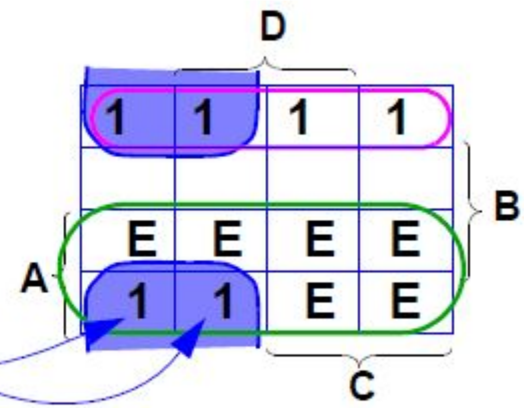
Example: F is true when:

both A and B are off
or both C and B are off
or both A and E are on



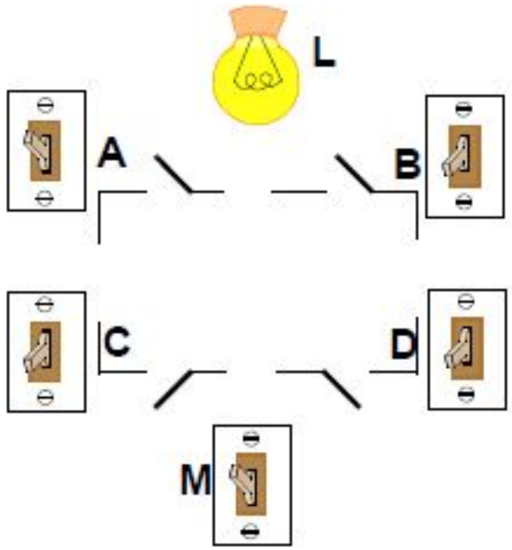


These squares not looped in $\overline{e}$ plane



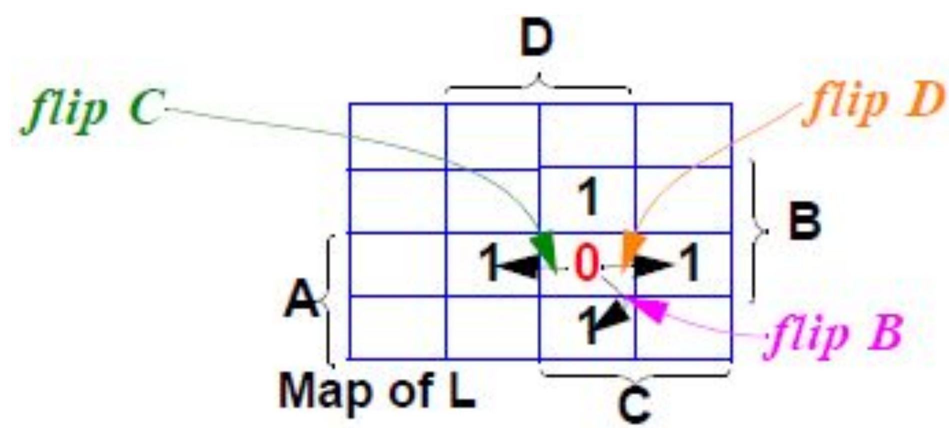
$F = EA +$	e plane
$\overline{A}\cdot\overline{B} + \overline{C}\cdot\overline{B} +$	both planes
0	$\overline{e}$ plane

Example: Light Control



Flicking A,B,C or D
-turns on light if it is off
-turns off light if it is on
M turns off all lights

Start at ABCD=1111; L= 0 (off)
Flick any one switch,
one bit will change.
Move one square, get L=1



If M = 0, light is always off.
All “1”s become “0”s.

Map of L

D

		1		1
	1		1	
A		1		1
A	1		1	

B

C

D

		M		M
	M		M	
A		M		M
A	M		M	

B

C

$$L = M(A \oplus B \oplus C \oplus D)$$

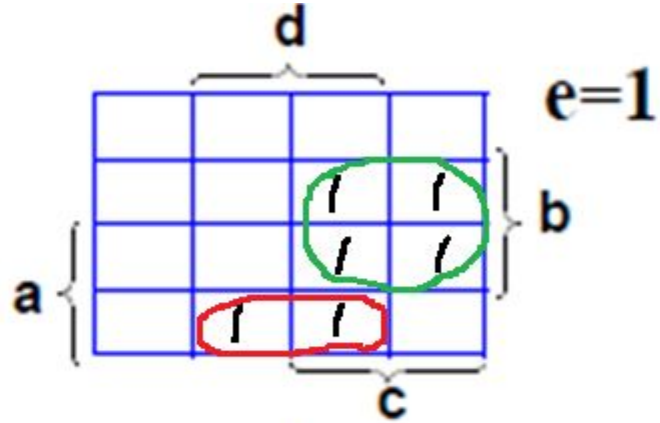
Changing any one bit changes L

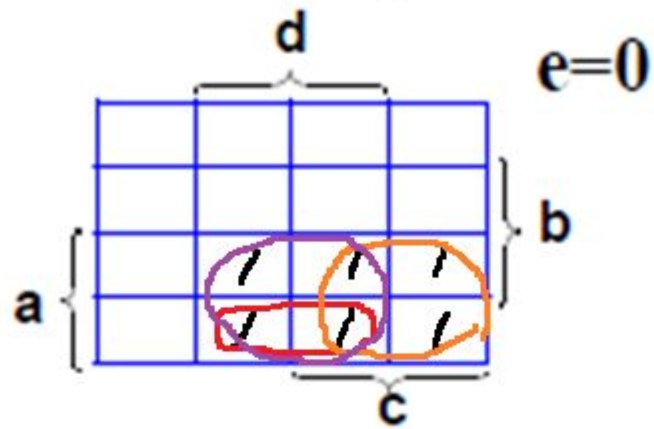
4-variable map is alternate “1”s and “0”s.

$$L = A \oplus B \oplus C \oplus D$$

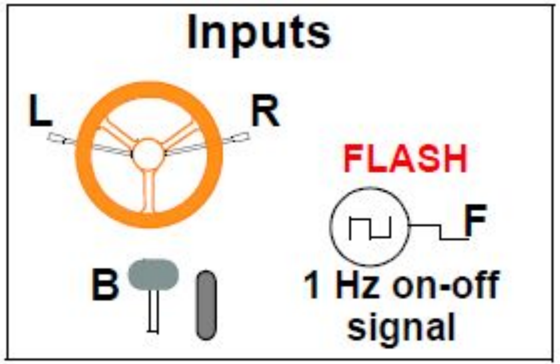
Example: $F = (abc + a\bar{b}d + cb)e + (a\bar{b}d + ac + adb)\bar{e}$

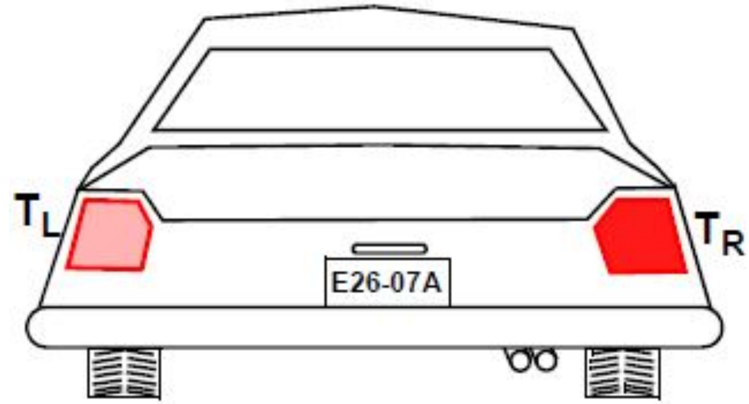
$= (abc + cb)e + (ac + adb)\bar{e} + a\bar{b}d$





Example: Tail Light Control





	Inputs LR B		Tail light T_L	Tail light T_R
- - -	00 0		0	0
- R -	01 0	Right turn	0	Flash
LR -	11 0	4-way,	Flash	Flash
L - -	10 0	Left turn,	Flash	0
- - B	00 1	Brake	Solid	Solid
- R B	01 1	Right turn & Brake	Solid	Flash
LR B	11 1	4-way and Brake	Flash	Flash
L - B	10 1	Left turn & Brake	Flash	Solid

LR	B	T_L	T_R
00	0	0	0
01	0	0	Flash
11	0	Flash	Flash
10	0	Flash	0
00	1	Solid	Solid
01	1	Solid	Flash
11	1	Flash	Flash
10	1	Flash	Solid

	B = 0	B = 1
		1
R		1
L, R	F	F
L	F	F

Tail light T_L

	B = 0	B = 1
		1
R	F	F
L, R	F	F
L		1

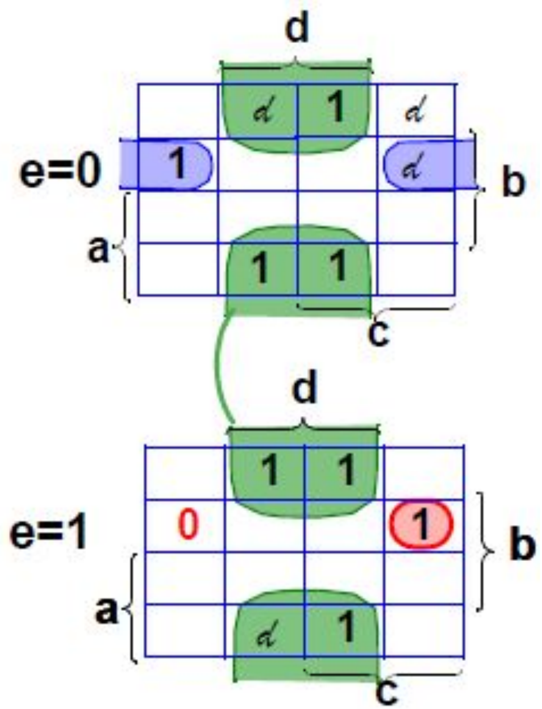
Tail light T_R

e=0

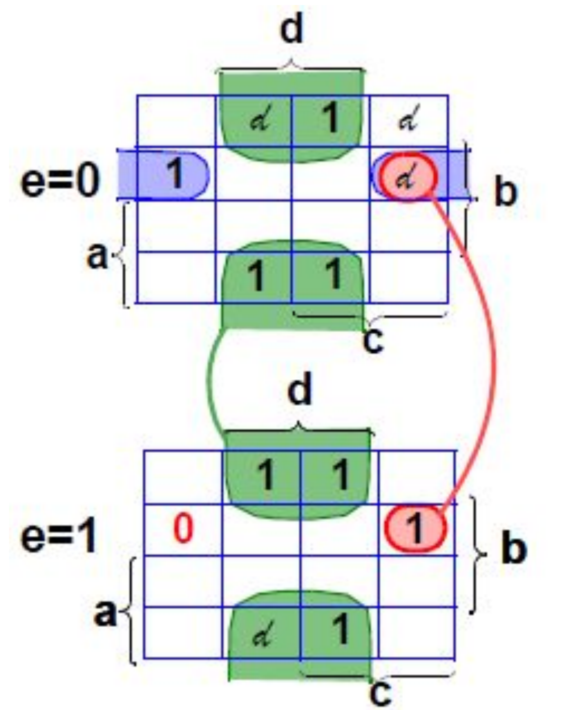
	d	1	d
1			d
	1	1	

e=1

	1	1	0
0			1
	d	1	



$$F = \bar{e}(\bar{a}b\bar{d}) + \bar{b}d + e(\bar{a}bc\bar{d})$$

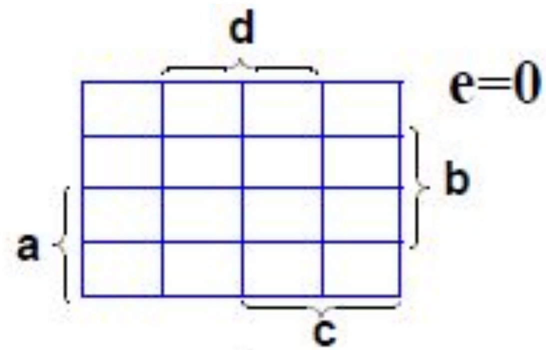


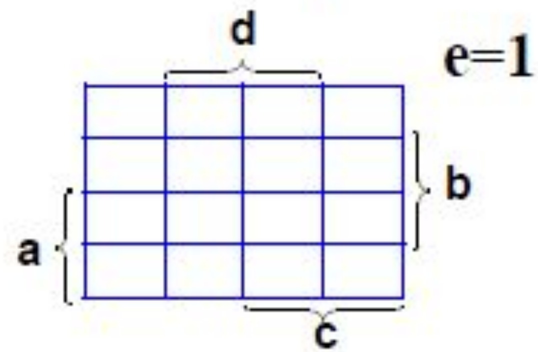
$$F = \bar{e}(\bar{a}b\bar{d}) + \bar{b}d + \bar{a}bc\bar{d}$$

Example: $F = a\bar{b}d + \bar{b}cd + \bar{a}b\bar{c}\cdot\bar{d}\cdot\bar{e} + \bar{a}bc\bar{d}e$

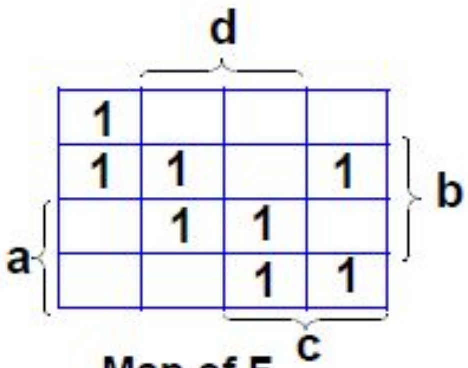
Further a,c,d,e can never take on the values 0,1,0,0 $(\bar{a}c\bar{d}\cdot\bar{e})$

and a,b,c,d can never take on the values 0,0,0,1 $(\bar{a}\cdot\bar{b}\cdot\bar{c}d)$

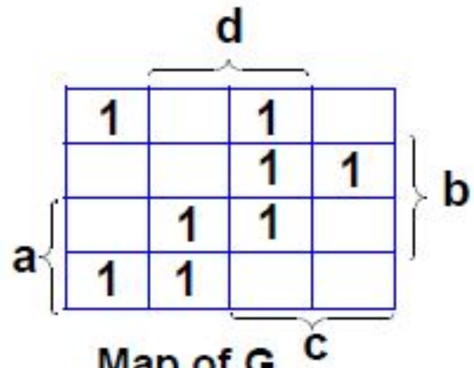




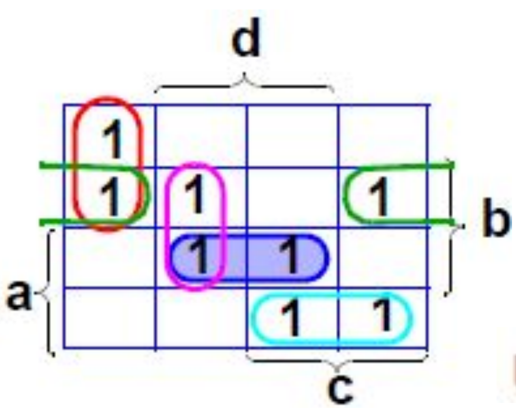
Example:



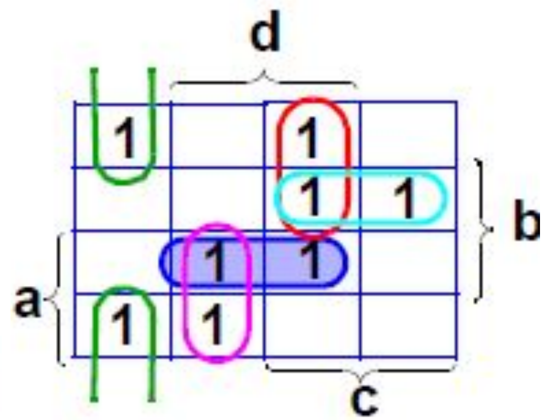
Map of F



Map of G

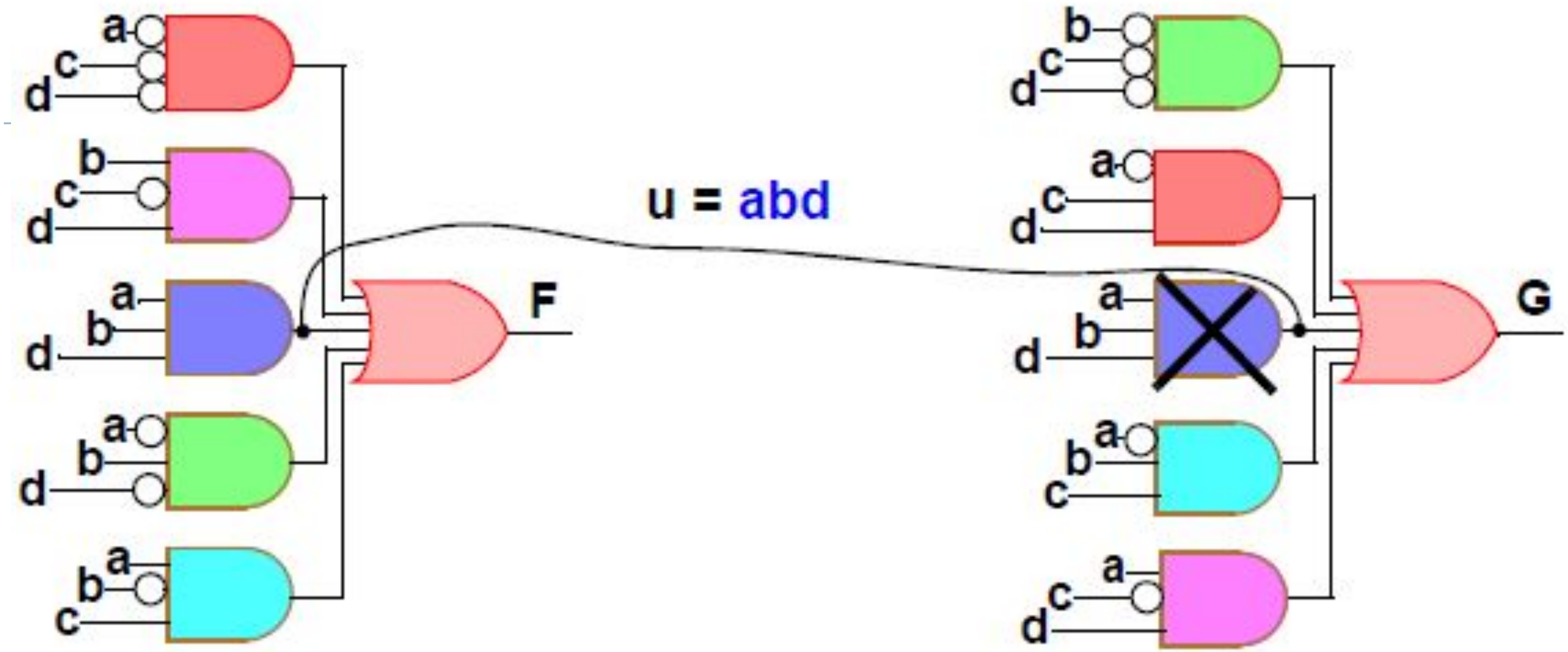


$u = abd$



$F = \bar{a}\cdot\bar{c}\cdot\bar{d} + b\bar{c}d$
$+u + \bar{a}b\bar{d} + a\bar{b}c$

$G = \bar{b}\cdot\bar{c}\cdot\bar{d} + \bar{a}cd$
$+u + \bar{a}bc + a\bar{c}d$

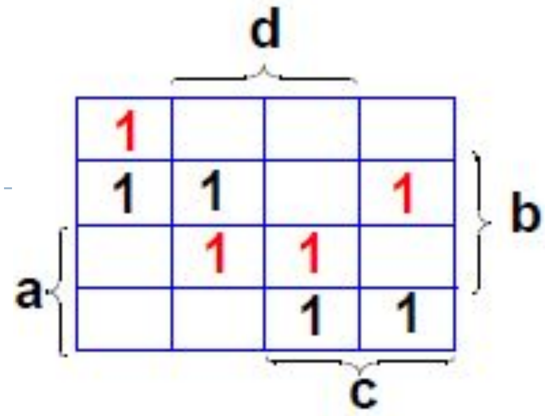


circuit size estimates

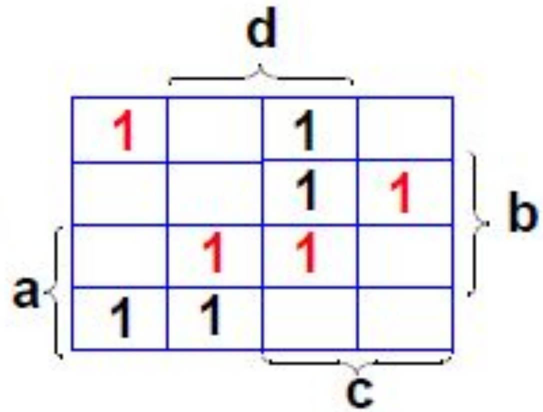
29 letters (literals) 13+13+3

37 gate inputs 15+5+12+5

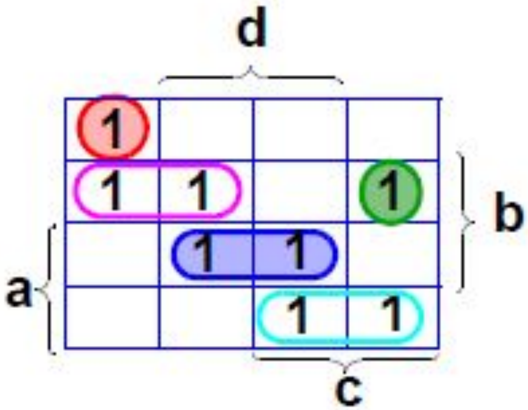
11 gates



Map of F



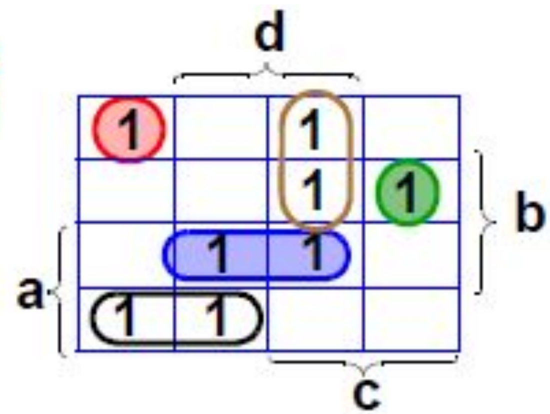
Map of G



$v=\bar{a}\cdot\bar{b}\cdot\bar{c}\cdot\bar{d}$

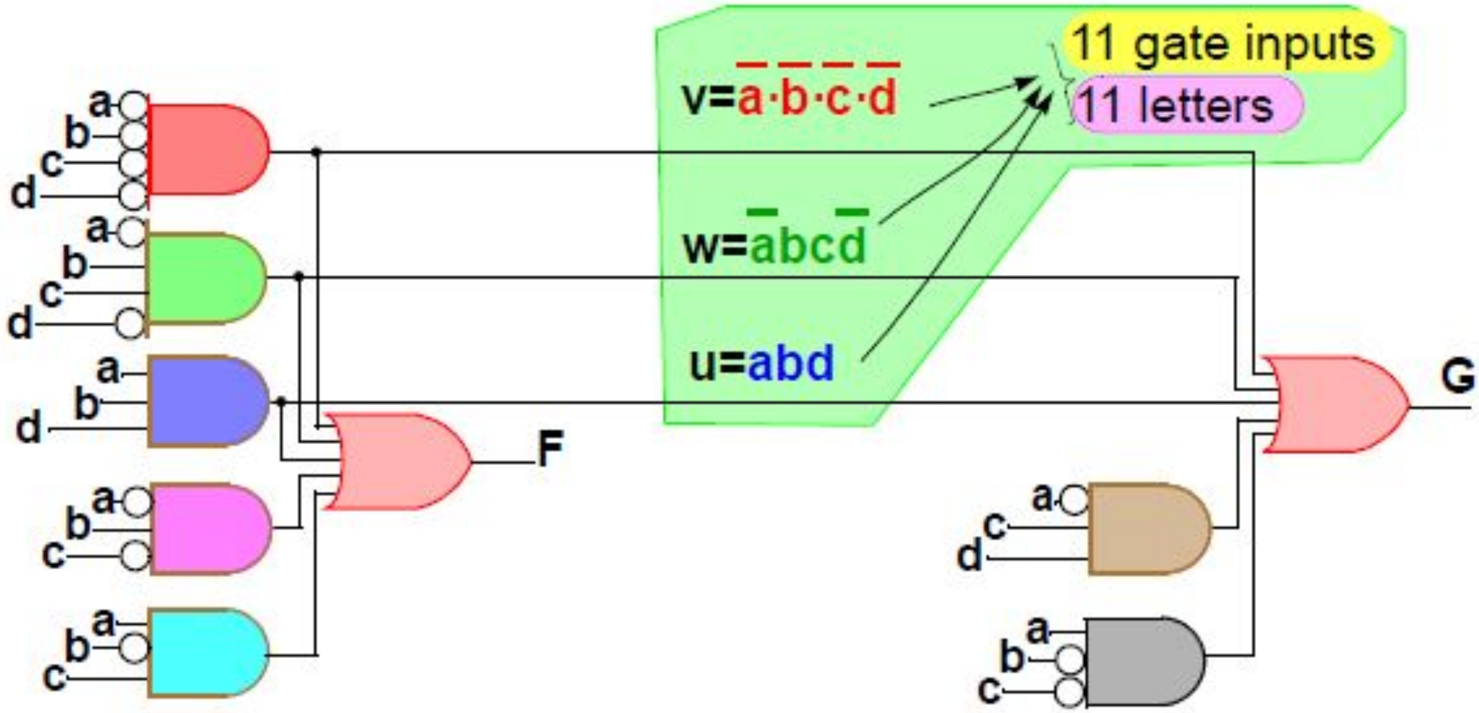
$w=\bar{a}bc\bar{d}$

$u=abd$



$F=v+w+u$
$\quad+\bar{a}b\bar{c}+a\bar{b}c$

$G=v+w+u$
$\quad+\bar{a}cd+a\bar{b}\cdot\bar{c}$



<u>size measures</u>

Prev slide	This slide	
29	29	letters (literals)
37	33	gate inputs
11	9	gates