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P1 Chapter 3

«Functions and combining functions»

A function is a **rule**, which calculates values of $f(x)$ for a set of values of x.

e.g. $f(x) = 2x - 1$ and $g(x) = \sin x$ are functions.

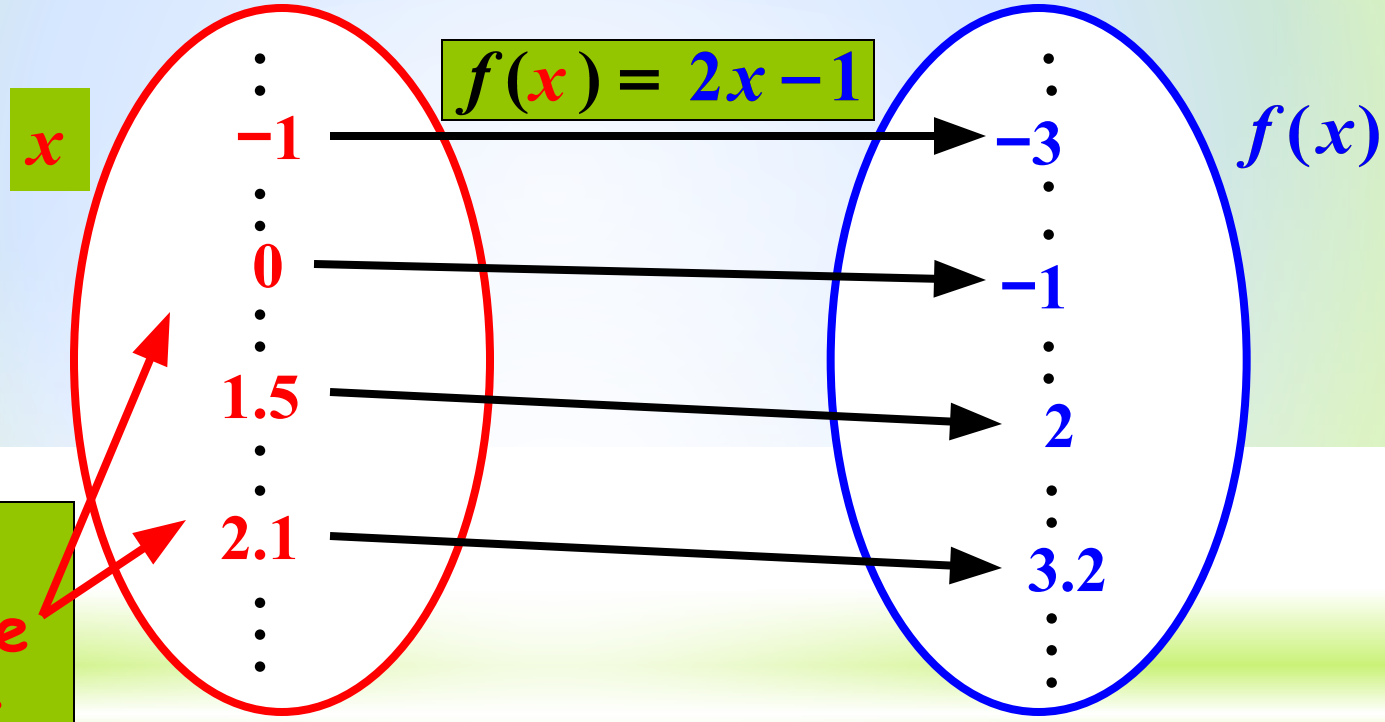
$f(x)$ is called the **image** of x

Another Notation

$f : x \mapsto 2x - 1$ means $f(x) = 2x - 1$

$f(x)$ is often replaced by y.

We can illustrate a function with a diagram



A few of the possible values of x

The rule is sometimes called a mapping.

A bit more jargon

To define a function fully, we need to know the values of x that can be used.

The set of values of x for which the function is defined is called the **domain**.

In the function $f(x) = x^2$ **any** value can be substituted for x, so the domain consists of

all real values of x

We write $x \in \mathbb{R}$

We say "real" values because there is a branch of mathematics known as complex numbers that we do not study.

$\mathbb{R}$ stands for the set of all real numbers

$\in$ means "belongs to."

So, $x \in \mathbb{R}$ means **x is any real number**

The **range** of a function $f(x)$ is the set of values given by $f(x)$.

e.g. Any value of x substituted into $f(x) = x^2$ gives a positive (or zero) value.

So the range of $f(x) = x^2$ is $f(x) \geq 0$

If $y = f(x)$, the range consists of the set of y-values, so

domain: x-values	range: y-values

Tip: To help remember which is the domain and which the range, notice that d comes before r in the alphabet and x comes before y.

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The set of values of x for which the function is defined is called the **domain**.

The **range** of a function is the set of values given by the rule.

domain: x-values	range: y-values

e.g. 1 Sketch the function $y = f(x)$ where $f(x) = x^2 + 4x - 1$ and write down its domain and range.

Solution: The quickest way to sketch this quadratic function is to find its vertex by completing the square.

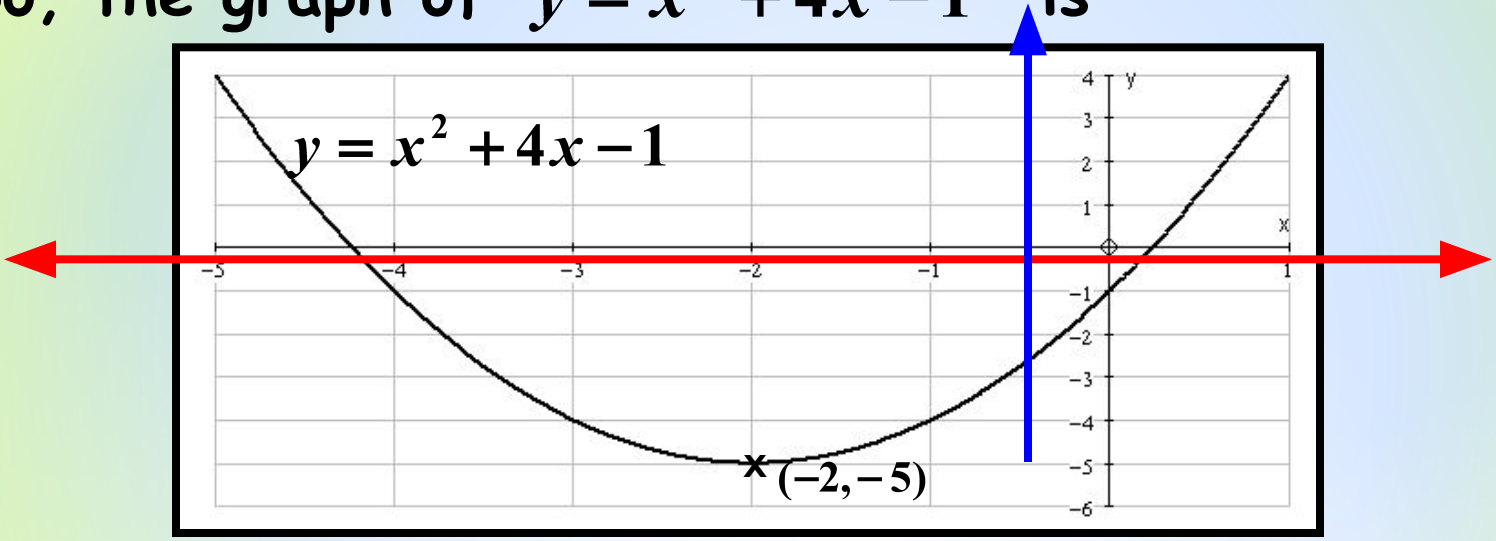
$$y = x^2 + 4x - 1 \quad \Rightarrow \quad y = (x+2)^2 - 4 - 1$$

$$\Rightarrow \quad y = (x+2)^2 - 5$$

This is a translation from $y = x^2$ of $\begin{bmatrix} -2 \\ -5 \end{bmatrix}$

so the vertex is $(-2, -5)$.

So, the graph of $y = x^2 + 4x - 1$ is



domain: The x-values on the part of the graph we've sketched go from −5 to +1 . . . BUT we could have drawn the sketch for <u>any</u> values of x.

So, we get $x \in \mathbb{R}$ (x is any real number)

BUT there are no y-values less than −5, . . .

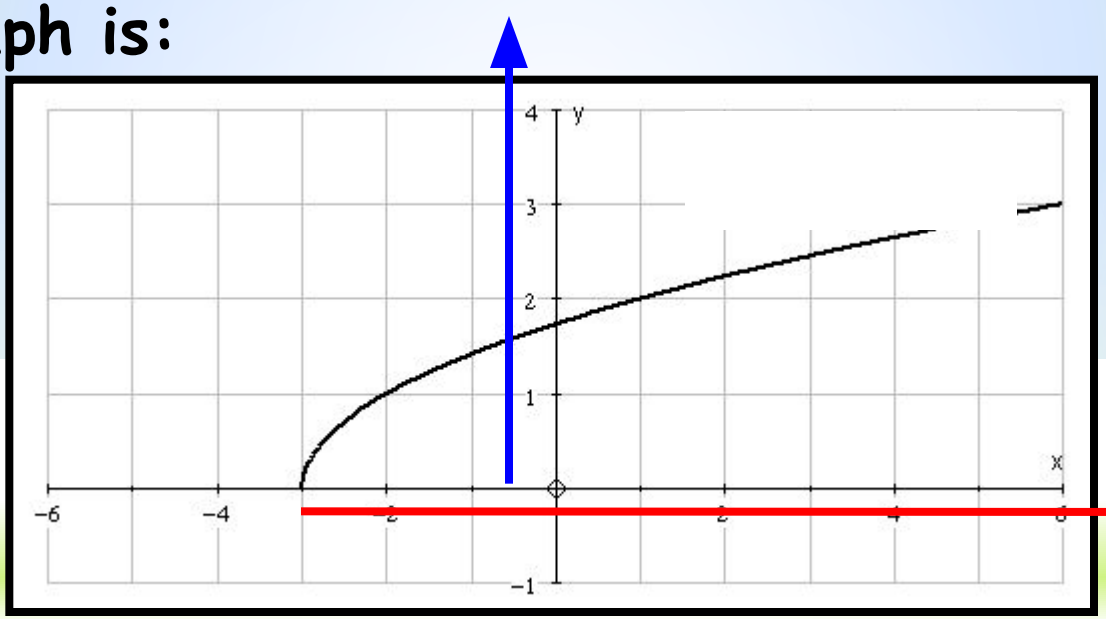
so the range is $y \geq -5$

(y is any real number greater than, or equal to, −5)

e.g.2 Sketch the function where
Hence find the domain and range of

Solution: $y = f(x)$ is a translation from of

so the graph is:





domain: x-values

$x \geq -3$

range: y-values

(We could write instead of y)

SUMMARY

- To define a function we need a rule and a set of values.
- Notation:

  means

- For ,

  the x-values form the domain

  the or y-values form the range

  e.g. For

  the domain is

  the range is



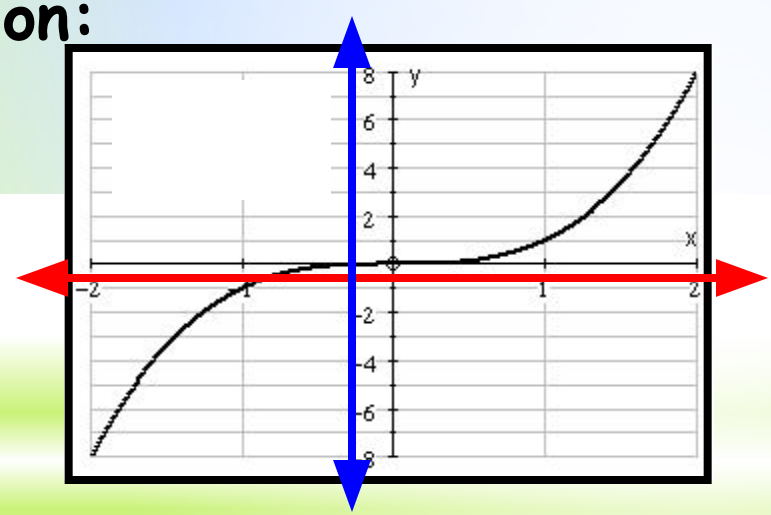
Exercise

1. Sketch the functions where

For each function write down the domain and range

Solution:

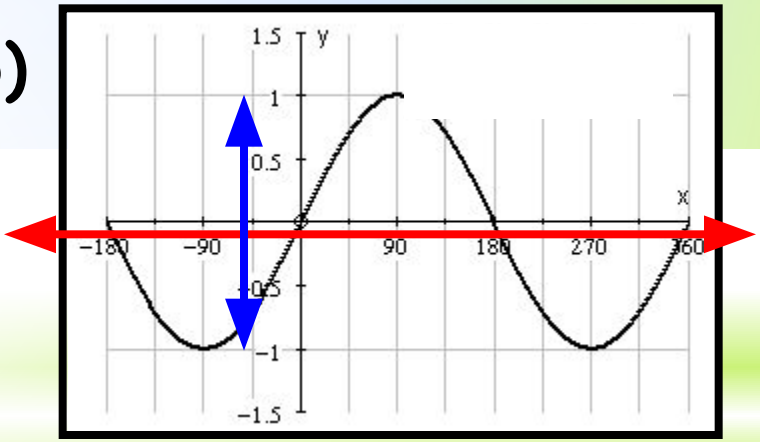
(a)



domain:

range:

(b)



domain:

range:

We can sometimes spot the domain and range of a function without a sketch.

e.g. For we notice that we can't square root a negative number (at least not if we want a real number answer) so,

$x + 3$ must be greater than or equal to zero.

So, the domain is

The smallest value of is zero.

Other values are greater than zero.

So, the range is

Functions of a Function

Suppose and

then, x is replaced by 3

Functions of a Function

Suppose and

then,

and

x is replaced by -1
x is replaced by

Functions of a Function

Suppose and

then,

and

We read as "f of g of x"

is "a function of a function" or compound function.

is the inner function and the outer.

x is "operated" on by the inner function first.

So, in we do g first.

Notation for a Function of a Function



 is often written as .

 does NOT mean multiply g by f.

When we meet this notation it is a good idea to change it to the full notation.

I'm going to write  always !

e.g. 1 Given that and find

Solution:

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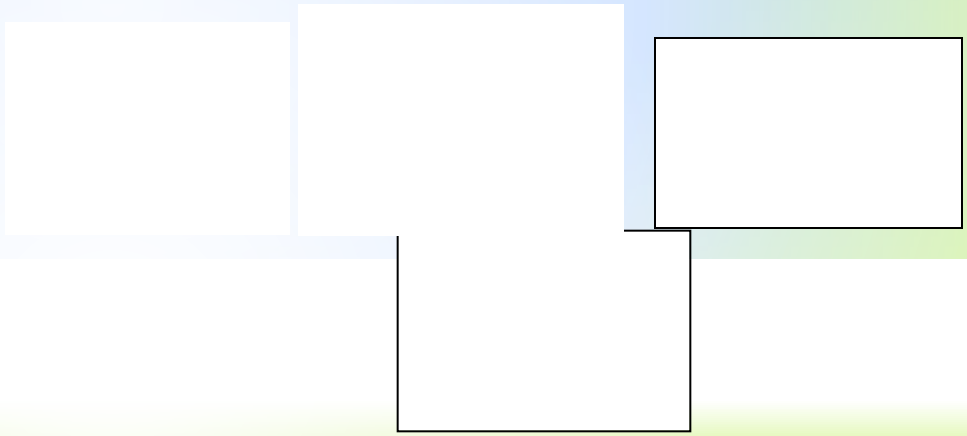
e.g. 1 Given that and find

Solution:

N.B. is not the same as

e.g. 1 Given that and find

Solution:



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Solution:

Exercise

1. The functions f and g are defined as follows:

R

(a) What is the range of f ?

(b) Find (i) and (ii)

Solution: (a) The range of f is

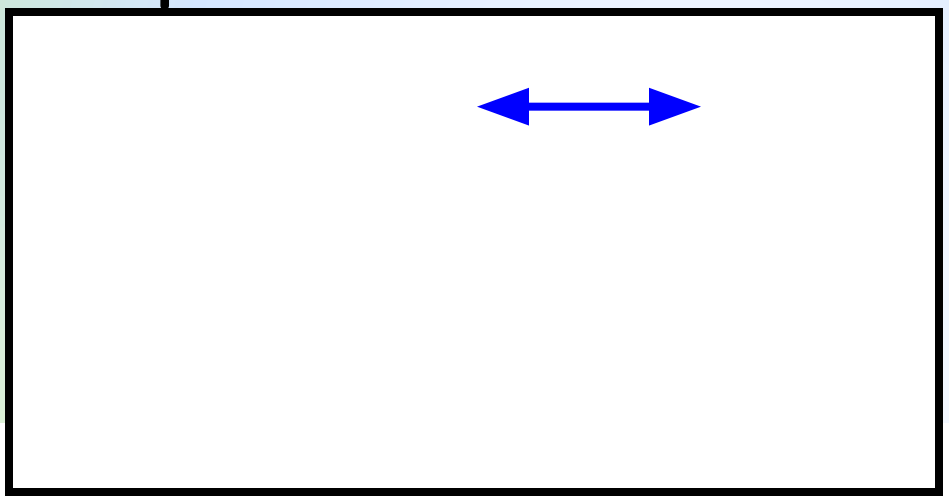
(b) (i)

(ii)

Periodic Functions

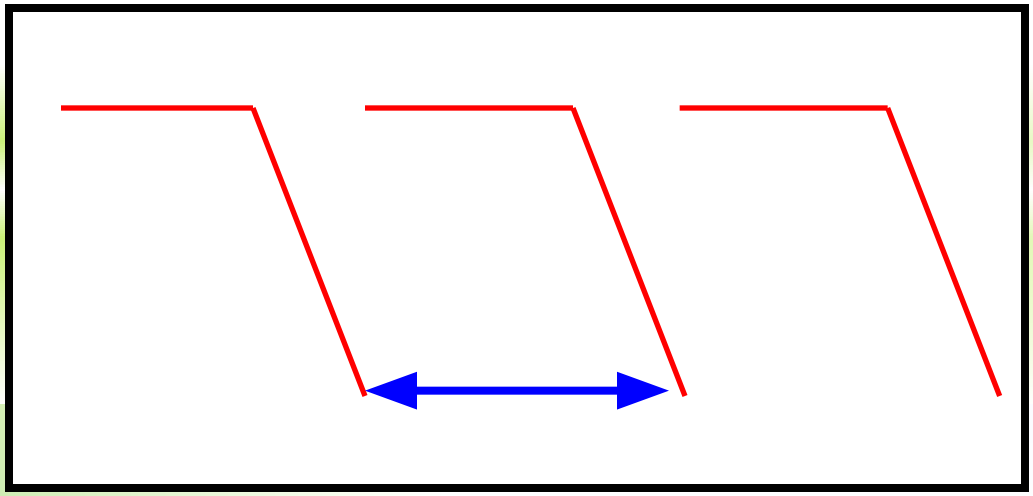
Functions whose graphs have sections which repeat are called periodic functions.

e.g.



peats every radian

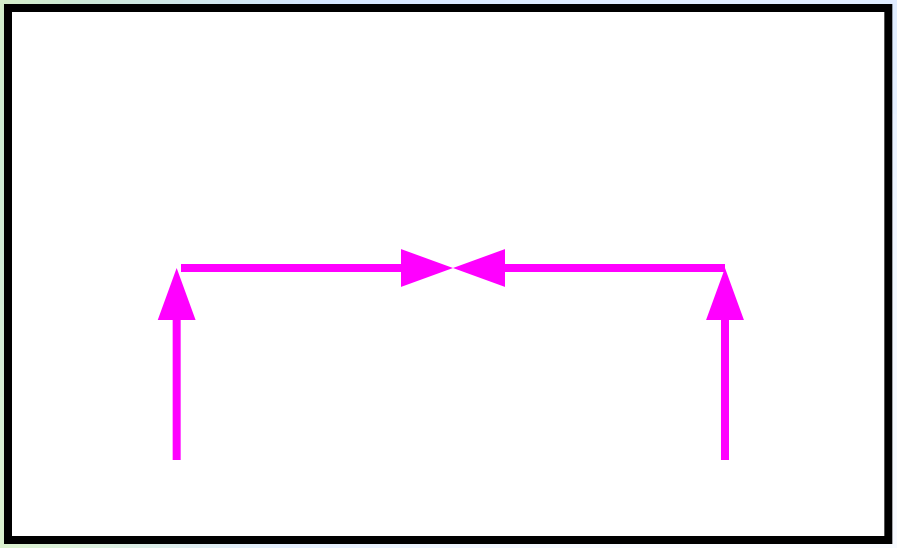
It has a period of



This has a period of 3.

Some functions are **even**

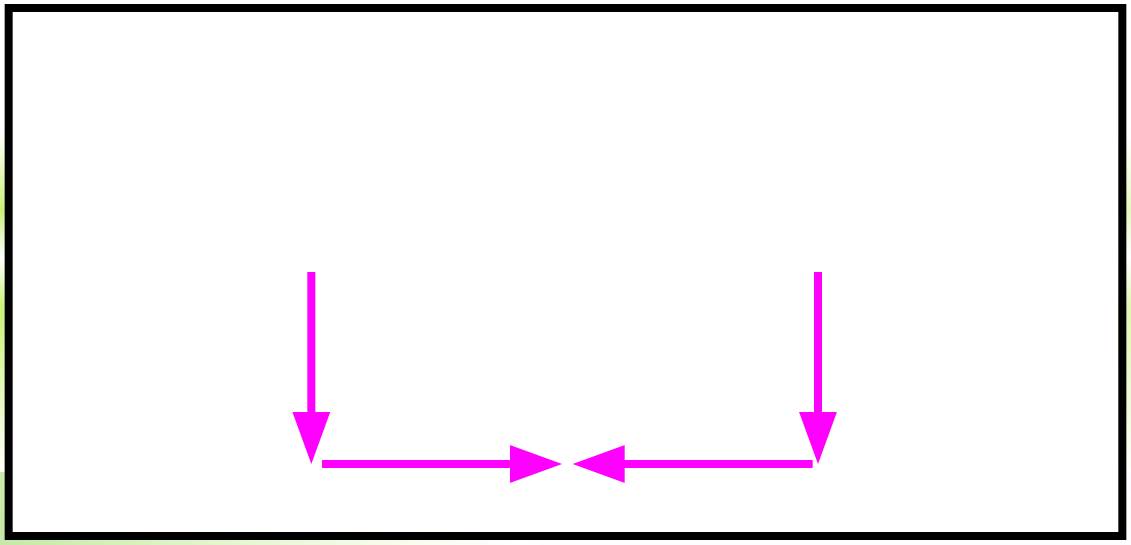
e.g.



Even functions are symmetrical about the y - axis

So,

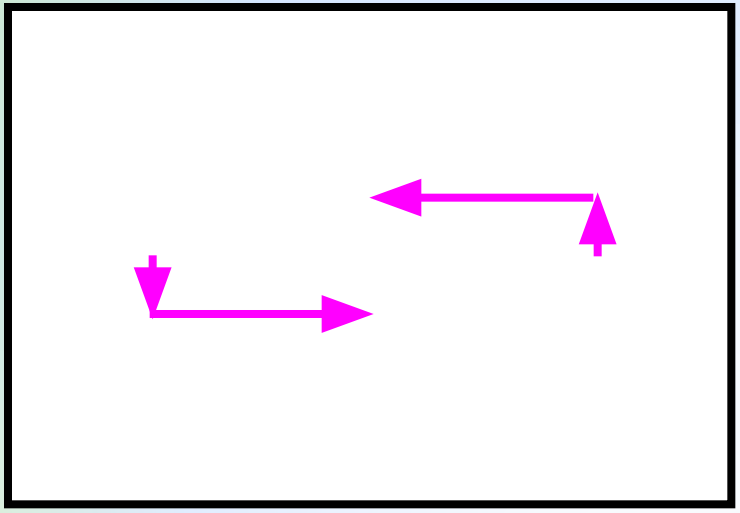
e.g.



e.g.

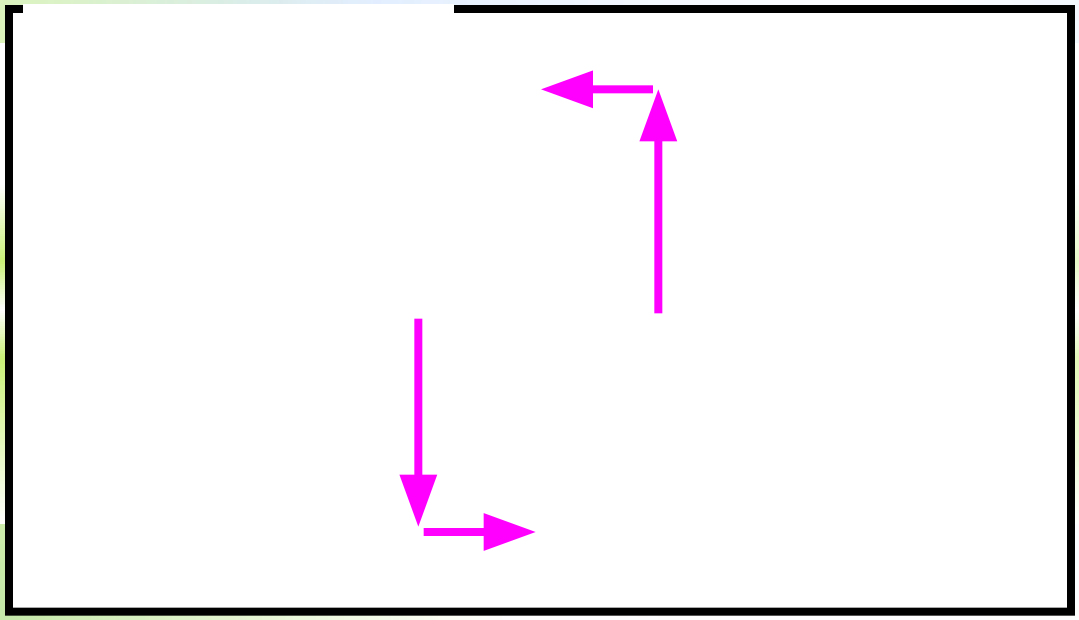
Others are odd

e.g.



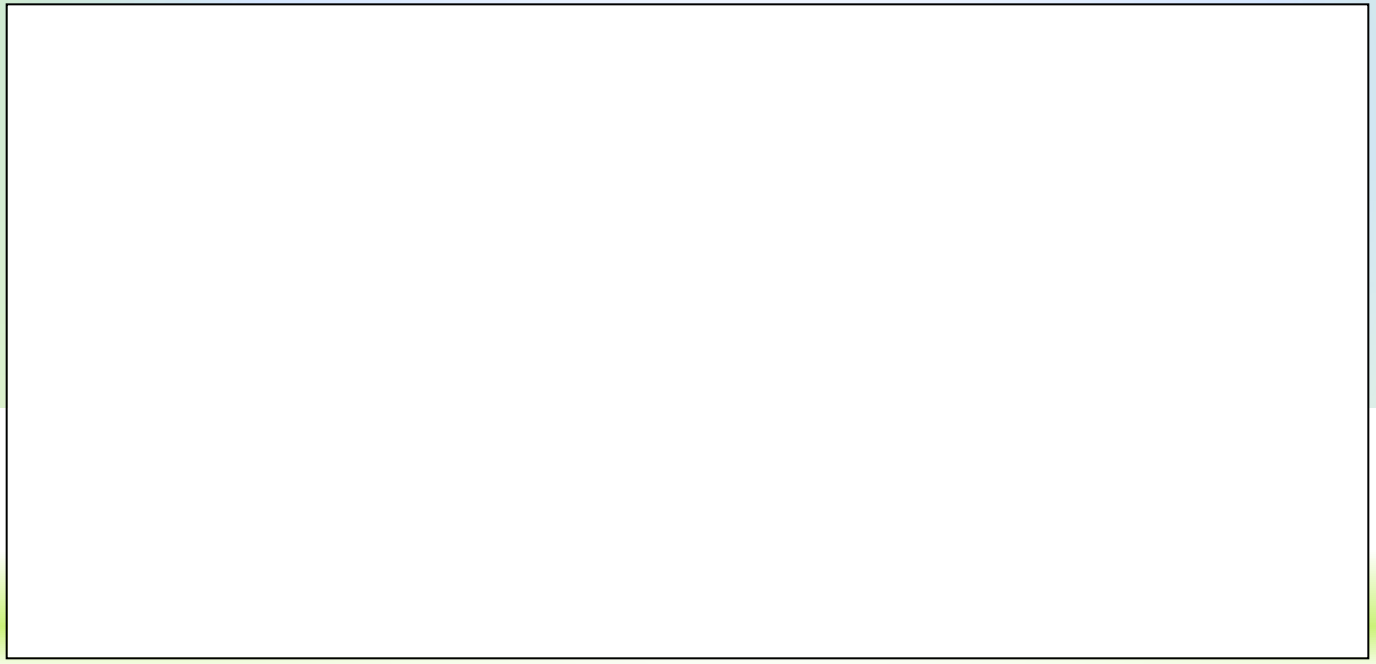
Odd functions have 180° rotational symmetry about the origin

e.g.



e.g.

Many functions are neither even nor odd
e.g.



Try to sketch one even function, one odd and one that is neither. Ask your partner to check.

SUMMARY

- A compound function is a function of a function.
- It can be written as which means
- The inner function is
- is not usually the same as
- is read as "f of g of x".